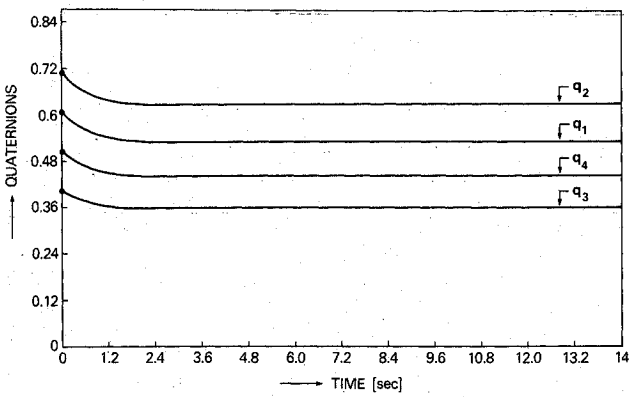

Fig. 3 Quaternions.

study the performance of the estimator in the presence of the feedback control that maintains the normalization. The following model, which follows Eq. (25), can be used for such purposes:

$$\dot{q} = \left[\tfrac{1}{2}\Omega(\omega) + K\left(\frac{1 - \|q\|}{\|q\|}\right)\right] q \tag{27}$$

$$\omega = g - b - \eta \tag{28}$$

where g is the gyro output and b is the gyro drift rate to be estimated along with q. η denotes the gyro noise. Equation (27) can be used for propagation, and the star tracker data can be utilized to identify the gyro drift and update q as it is proposed in Ref. 6.

VI. Summary

Quaternion normalization has been successfully carried out using optimal control techniques. A matrix Riccati equation results for the computation of the gain matrix. Simulation results indicate that a high precision of the order of 10^{-12} can be achieved by this technique in meeting the constraint $q^T q = 1$.

The effect of normalization on the update equations needs further study. The modified model given in Eq. (27) can be utilized for this purpose. The scheme can be implemented without increasing very much the computer loading because K an be precomputed and stored.

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Book Announcements

CHANKONG, V., Khon Kaen University, and **HAIMES, Y.Y.,** Case Western Reserve University, ***Multiobjective Decision Making:* Theory and Methodology,** North-Holland, New York, 1983, 406 pages. $45.00.

Purpose: This book is intended for graduate students, researchers, and engineers interested in solving multiobjective optimization problems. Undergraduate-level understanding of set theory, matrices, probability, linear programming, and mathematic analysis is required.

Contents: Elements of multiobjective decision problems. Fundamentals: selected background topics. Utility theory. Vector optimization theory. Assessment methodologies. Methods for generating noninferior solutions. Noninteractive and interactive multiobjective progamming methods. The surrogate worth trade-off method and its extensions. Comparative evaluation and comments. Appendix. Author index. Subject index.

DOWNTON, A.C., University of Southampton, ***Computers and Microprocessors Components and Systems,*** Van Nostrand Reinhold (UK) Co. Ltd., 1984, 182 pages. $11.50.

Purpose: This book is intended as a beginning text for a lower-level undergraduate course and, as such, may be of interest to engineers who want to learn something about the details of computer operation. The goal of the text is to present an integrated approach to computers and microprocessors which places equal emphasis on components and systems, application and design.

Contents: Computers and microprocessors. Memory structure and architecture. Data representation in computers. Data processing. Input and output interfaces. Instruction size and addressing modes. Programming computers and microprocessors. Computer systems. System software. Microprocessor application design example. Appendixes. References. Answers to exercises and problems. Index.